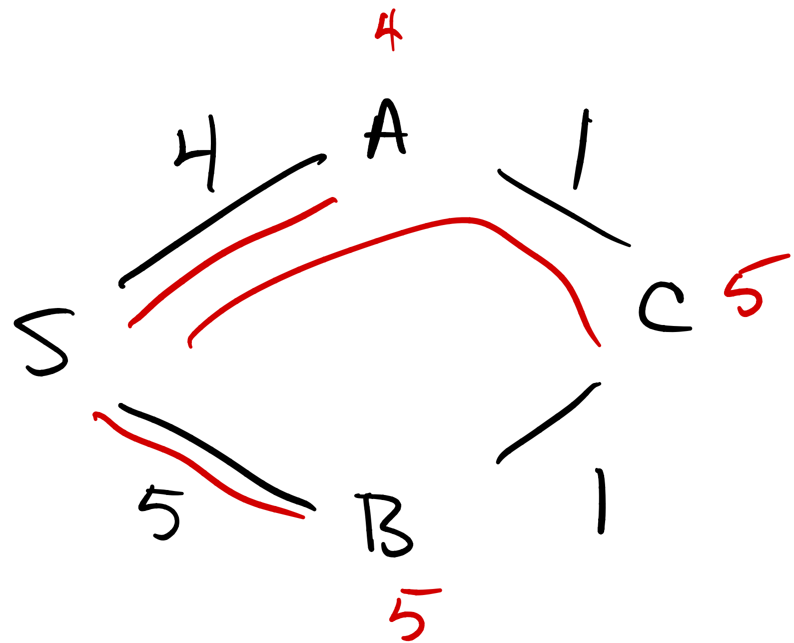
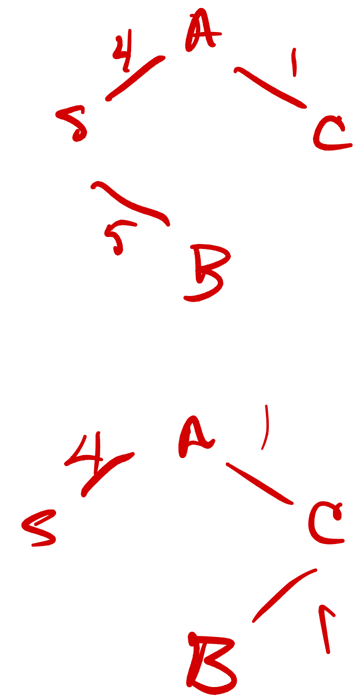
CS 252

M, 13 May 2024

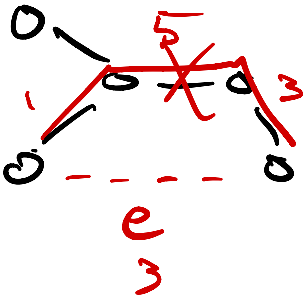
8c




DA.



7.




⓪ Given $e = (u, v)$ with weight $w$

① BFS in $T$ from $u$ to find shortest path $O(|V|)$

$u = u_1, u_2, \ldots, u_k = v$

② Find max edge weight in that path $O(|V|)$

$(u_i, u_{i+1})$ has weight $w'$

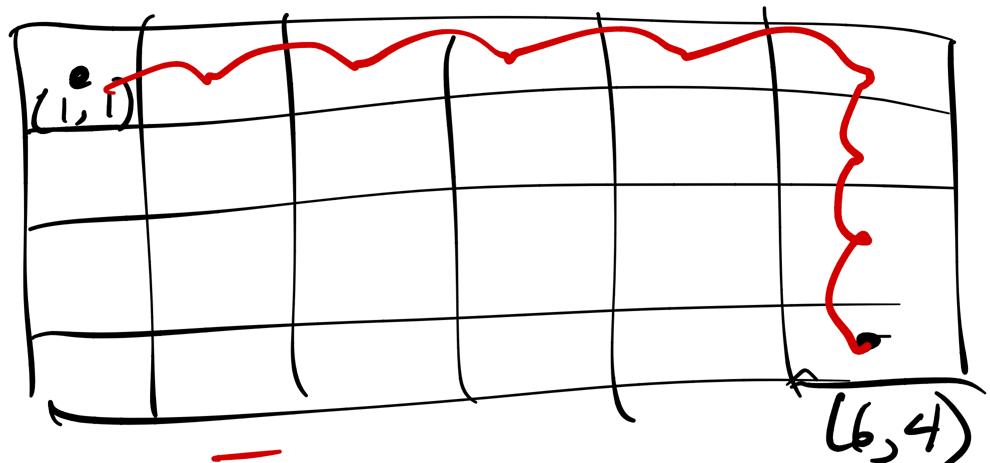
③ if $w' > w$, delete $(u_i, u_{i+1})$ + add $(u, v)$ $O(1)$

otherwise, leave $e$ out

$M(6,4)$

6

4

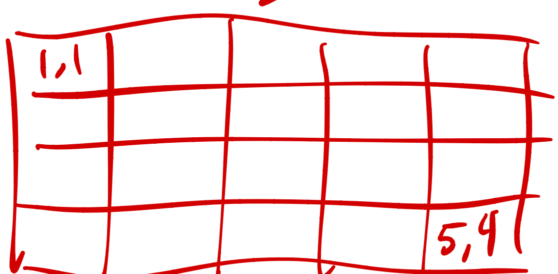



Walk (1,1) to (6,4) only R + Down

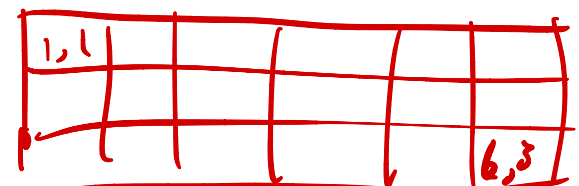
How many such routes

5

4




$M(5,4)$




$M(6,3)$

| | size | value |
|---|---|---|
| 0 | ☒ | ☒ |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

$d$

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | | | | | | | | | | | | |
| 2 | 0 | | | | | | | | | | | | |
| 3 | 0 | | | | | | | | | | | | |
| 4 | 0 | | | | | | | | | | | | |
| 5 | 0 | | | | | | | | | | | | |
| 6 | 0 | | | | | | | | | | | | |
| 7 | 0 | | | | | | | | | | | | |
| 8 | 0 | | | | | | | | | | | | |

$$M(k, d) = \max \begin{cases} \text{value}[k] + M(k-1, d - \text{size}[k]) & \text{if } k > 0,\ d \geq \text{size}[k] \\ M(k-1, d) & \text{if } k > 0,\ d \geq 0 \\ 0 & \end{cases}$$

| | size | value |
|---|---|---|
| 0 | X | X |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

$d$

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | | | | | | | | | | | |
| 3 | 0 | 1 | | | | | | | | | | | |
| 4 | 0 | 2 | | | | | | | | | | | |
| 5 | 0 | 2 | | | | | | | | | | | |
| 6 | 0 | 9 | | | | | | | | | | | |
| 7 | 0 | 9 | | | | | | | | | | | |
| 8 | 0 | 9 | | | | | | | | | | | |

$$M(k, d) = \max \begin{cases} value[k] + M(k-1, \; d - size[k]) & \text{if } k > 0, d \geq size[k] \\ M(k-1, d) & \text{if } k > 0, d \geq 0 \\ 0 & \end{cases}$$

| | size | value |
|---|---|---|
| 0 | X | X |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

d

| k \ d | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 13 | 13 | 13 | 13 | 13 | 13 |
| 3 | 0 | 1 | | | | | | | | | | | |
| 4 | 0 | 2 | | | | | | | | | | | |
| 5 | 0 | 2 | | | | | | | | | | | |
| 6 | 0 | 9 | | | | | | | | | | | |
| 7 | 0 | 9 | | | | | | | | | | | |
| 8 | 0 | 9 | | | | | | | | | | | $\boxed{\phantom{0}}$ |

goal answer

$$M(k, d) = \max \begin{cases} value[k] + M(k-1, \quad - size[k]) & \text{if } k > 0,\ d \geq size[k] \\ M(k-1, d) & \text{if } k > 0,\ d \geq 0 \\ 0 & \end{cases}$$

Algorithm $M(n, C)$

```
O(n)   for k = 0 to n
           M[k, 0] = 0

O(C)   for d = 0 to C
           M[0, d] = 0

O(nC)  for k = 1 to n
           for d = 1 to C
               m1 = value[k] + M[k-1, d - size[k]] if d >= size[k]
                                                   else 0
               m2 = M[k-1, d]
               M[k, d] = max(m1, m2)
```

| | size | value |
|---|---|---|
| 0 | X | X |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

d

| k \ d | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 13 | 13 | 13 | 13 | 13 | 13 |
| 3 | 0 | 1 | 7 | 8 | 8 | 8 | 8 | 13 | 14 | 14 | 14 | 14 | 14 |
| 4 | 0 | 2 | 7 | 9 | 10 | 10 | 10 | 13 | 15 | 16 | 16 | 16 | 16 |
| 5 | 0 | 2 | 7 | 9 | 10 | 11 | 13 | 14 | $\boxed{15}$ | | | | |
| 6 | 0 | 9 | | | | | | | | | | | |
| 7 | 0 | 9 | | | | | | | | | | | |
| 8 | 0 | 9 | | | | | | | | | | | |

$$M(k,d) = \max \begin{cases} value[k] + M(\overset{4}{k-1}, \; \underbrace{\quad}_{4} - \underbrace{size[k]}_{3}) & \text{if } k>0,\ d \geq size[k] \\ M(k-1, d) & \text{if } k>0,\ d \geq 0 \\ 0 \end{cases}$$

10

$4+10=14 < 15$

| | size | value |
|---|---|---|
| 0 | ☒ | ☒ |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

$d$

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 13 | 13 | 13 | 13 | 13 | 13 |
| 3 | 0 | 1 | 7 | 8 | 8 | 8 | 8 | 13 | 14 | 14 | 14 | 14 | 14 |
| 4 | 0 | 2 | 7 | 9 | 10 | 10 | 10 | 13 | 15 | 16 | 16 | 16 | 16 |
| 5 | 0 | 2 | 7 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | 17 | | |
| 6 | 0 | 9 | | | | | | | | | | | |
| 7 | 0 | 9 | | | | | | | | | | | |
| 8 | 0 | 9 | | | | | | | | | | | |

$$M(k,d) = \max \begin{cases} value[k] + M(k-1, d - size[k]) & \text{if } k > 0,\ d \geq size[k] \\ M(k-1, d) & \text{if } k > 0,\ d \geq 0 \\ 0 & \end{cases}$$

| | size | value |
|---|---|---|
| 0 | | |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

d

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 13 | 13 | 13 | 13 | 13 | 13 |
| 3 | 0 | 1 | 7 | 8 | 8 | 8 | 8 | 13 | 14 | 14 | 14 | 14 | 14 |
| 4 | 0 | 2 | 7 | 9 | 10 | 10 | 10 | 13 | 15 | 16 | 16 | 16 | 16 |
| 5 | 0 | 2 | 7 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | | | |
| 6 | 0 | 9 | | | | | | | | | | | |
| 7 | 0 | 9 | | | | | | | | | | | |
| 8 | 0 | 9 | | | | | | | | | | | |

$$M(k, d) = \max \begin{cases} value[k] + M(k-1, \quad - size[k]) & \text{if } k > 0, d \geq size[k] \\ M(k-1, d) & \text{if } k > 0, d \geq 0 \\ 0 & \end{cases}$$

| | size | value |
|---|---|---|
| 0 | ☒ | ☒ |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

d

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 0 | 0 | 7 | 7 | 7 | 7 | 7 | 13 | 13 | 13 | 13 | 13 | 13 |
| 3 | 0 | 1 | 7 | 8 | 8 | 8 | 8 | 13 | 14 | 14 | 14 | 14 | 14 |
| 4 | 0 | 2 | 7 | 9 | 10 | 10 | 10 | 13 | 15 | 16 | 16 | 16 | 16 |
| 5 | 0 | 2 | 7 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | 17 | 19 | 20 |
| 6 | 0 | 9 | 11 | 16 | 18 | 19 | 20 | 22 | 23 | 24 | 25 | 26 | 28 |
| 7 | 0 | 9 | 11 | 16 | 18 | 19 | 20 | 22 | 23 | 24 | 25 | 26 | 28 |
| 8 | 0 | 9 | 11 | 16 | 18 | 19 | 20 | 22 | 23 | 24 | 25 | 26 | 28 |

Back-tracing

How did I get here?

$$M(k,d) = \max \begin{cases} value[k] + M(k-1, \quad - size[k]) & \text{if } k>0,\ d \geq size[k] \\ M(k-1, d) & \text{if } k>0,\ d \geq 0 \\ 0 \end{cases}$$

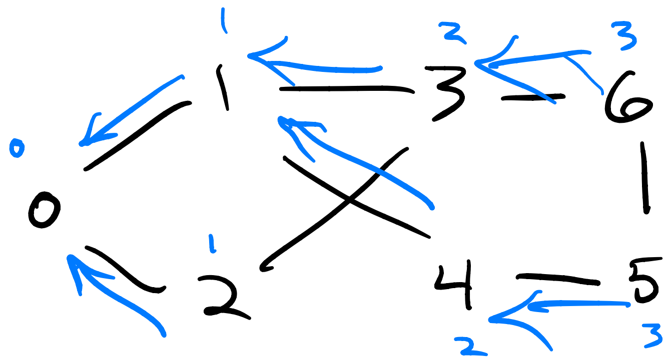
# Backtracing

- Problems: maximize/minimize $F(s)$
  over all $s \in S$ (search space)
- Algorithm: finds best value of $F(s)$
- Question: can you also get $m \in S$
  such that $F(m)$ is optimal
  (not just the optimal value, but
  also the way we get it)

# Backtracing

- BFS: got v's distance from start node u, but what's the path from u to v?
- Dijkstra: same
- Wire-cutting: best profit, but where should I make the cuts?
- Knapsack: best total value, but which objects should I select?

# BFS




Queue ~~1~~ ~~2~~ 3 4

parent? predecessor?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -1 | 0 | 0 | 1 | 1 | 4 | 3 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | 8 | 10 | 12 | 16 |
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

$$M(n) = \max_{i=1,\dots,n} \left( p[i] + M(n-i) \right)$$

$$M(0) = 0$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| M | 0 | 2 | 4 | 8 | 10 | 12 | 16 |
| p | 0 | 2 | 3 | 8 | 9 | 12 | 13 |

| | size | value |
|---|---|---|
| 0 | ☒ | ☒ |
| 1 | 2 | 7 |
| 2 | 5 | 6 |
| 3 | 1 | 1 |
| 4 | 1 | 2 |
| 5 | 3 | 4 |
| 6 | 1 | 9 |
| 7 | 8 | 10 |
| 8 | 6 | 6 |

d

| k | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | | | | | | | | | | | | |
| 2 | 0 | | | | | | | | | | | | |
| 3 | 0 | | | | | | | | | | | | |
| 4 | 0 | | | | | | | | | | | | |
| 5 | 0 | | | | | | | | | | | | |
| 6 | 0 | | | | | | | | | | | | |
| 7 | 0 | | | | | | | | | | | | |
| 8 | 0 | | | | | | | | | | | | |

$$M(k, d) = \max \begin{cases} value[k] + M(k-1, \quad - size[k]) & \text{if } k > 0, d \geq size[k] \\ M(k-1, d) & \text{if } k > 0, d \geq 0 \\ 0 & \end{cases}$$