Connecting Discrete Mathematics and Computer Science

David Liben-Nowell

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Connecting Discrete Mathematics and Computer Science

REVISED EDITION

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Department of Computer Science

Carleton College

CAMBRIDGE UNIVERSITY PRESS

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To MDSWM, with never-ending appreciation, and in loving memory of my grandfather, Jay Liben, who brought more joy, curiosity, and kvetching to this world than anyone else I know.

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Would thou hadst less deserved,
That the proportion both of thanks and payment
Might have been mine! only I have left to say,
More is thy due than more than all can pay.

William Shakespeare (1564–1616)
The Scottish Play (c. 1606)

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David Liben-Nowell
Northfield, MN
August 2021

PS: I would be delighted to receive any comments or suggestions. Please don't hesitate to get in touch.

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Credits

This book is a revised version of a book first published as *Discrete Mathematics for Computer Science* in 2017 (with a preliminary edition published in 2015) by John Wiley & Sons.

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