Math 4990 Problem Set 6

Due Tuesday, Oct 14, 2014 in class

Errata

p.65, Unsolved Problem 11, “exponential number of triangles triangulations”

p.68, last line, “by our induction hypothesis establishes the theorem.”

Assignment

Liberally peruse pages 64–69, 98–102 of [DO].

[DO] Exercises 3.19, 3.20 (for \( n \geq 4 \)), 4.4, and 4.5 (“simple” means at most a few sentences).

Problem 5. Let \( G \) be a triangulation graph and \( a, b, c \) three of its vertices. Show that \( G \) has a vertex \( v \) distinct from \( a, b, c \) such that the degree of \( v \) is at most five.

Note that this is a strengthening of Exercise 3.14 we used in class for the proof of Fáry theorem.

Problem 6. Recall that the number of triangulations of an \( n + 2 \)-gon is the Catalan number \( C_n \). For infinitely many values of \( n \), construct a point set \( S \) with \( n + 2 \) points such that the number of triangulations of \( S \) is greater than \( C_n \). (See Exercises 3.15 and 3.18.)

Note that “for infinitely many values of \( n \)” is a phrase mathematicians use when they want something more general than \( n = 23 \), say, but do not need it for every single value of \( n \). For example, perhaps your construction works only for even \( n \) greater than 42, prime numbers, or \( n \) such that its proper positive integer divisors sum to itself. We refer to these as “infinite families of counterexamples.”