Math 4707 Midterm 3 Practice Questions

Assume that all graphs are simple.

Problem 1. Let P be a convex polyhedron in which every face is a triangle. Let $G = (V, E)$ be the skeleton of P. Prove that
$$\sum_{v \in V}(6 - d(v)) = 12.$$

Problem 2. Let G be a connected, planar, bipartite, simple graph with at least 3 edges and no cycles of length 4. Fix an embedding of G and let v, e, and f denote the number of vertices, edges, and faces, respectively.

(a) Prove that $f \leq \frac{e}{3}$.
(b) Prove that $e \leq \frac{3}{2}v - 3$.

Problem 3. Prove that there is a way to colour the edges of K_n red or blue such that there are at most
$$\binom{n}{a} 2^{1-\binom{a}{2}}$$
monochromatic K_a subgraphs.

Problem 4. Let G be a convex n-gon such that no three diagonals intersect at the same point. Prove that the number of regions divided by the sides and the diagonals of G is $\binom{n}{4} + \frac{n(n-3)}{2} + 2$.

Problem 5. Count the number of proper k-colourings for some small graphs. For example, the graph shown below.