Math 4707 Midterm 2 Practice Questions

Assume that all graphs are simple.


(a) Could the vertices of $G$ have degrees 1, 3, 4, 4, 5, 5, 5?
(b) Could the vertices of $G$ have degrees 0, 1, 3, 4, 4, 4, 6?
(c) How many 2-regular graphs (every vertex has degree 2) on vertex set $[7]$ are there?

Problem 2. Suppose $G$ is a graph with exactly two vertices $u$ and $v$ of odd degree. ($G$ may have vertices of even degrees.) Prove or disprove the following.

(a) $G$ is connected.
(b) There is a path from $u$ to $v$.

Problem 3. Recall that there are $n^{n-2}$ (labelled) trees on $[n]$.

(a) What is the number of trees on $[n]$ where 1 is a leaf?
(b) What is the number of trees on $[n]$ where 1 has degree 2?

Problem 4. Suppose $G$ is a forest on 100 vertices and 70 edges. How many connected components could $G$ have?

Problem 5. A chord of a cycle is an edge connecting two non-adjacent vertices of the cycle. Show that if every vertex of $G$ has degree at least 4, then $G$ has a cycle with two (or more) chords.

Problem 6. Let $G$ be a connected graph on $n$ vertices and $n$ edges, $n \geq 3$.

(a) Show that $G$ can be obtained from a tree with $n$ vertices by adding a new edge.
(b) Show that the number of spanning trees of $G$ is at least 3 and at most $n$.

Problem 7. An $n$-wheel is the graph obtained from an $n$-cycle by adding a new vertex that is adjacent to all $n$ vertices of the cycle. (Thus an $n$-wheel has $n + 1$ vertices and $2n$ edges.)

(a) Find the number of perfect matchings of an $n$-wheel.
(b) Find the number of matchings of an $n$-wheel.

Problem 8. Show that a graph is bipartite if and only if it has no odd cycles.