1. INTRODUCTION

Lecture 1
- Instructor: Calin Martin.
- Location: MOORE 100.

Sections 1E and 1F
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- SMC: Sept. 30–Dec. 3, M–R 09:00–15:00, MS 3974, M 13:00–14:00.

2. ADMINISTRATION

- HW due Fridays in lecture, and I will hand back in section.
- Determine office hour.

3. PRECALCULUS REVIEW

3.1. Exercise 1.2.21. Find the equation of the perpendicular bisector of the segment joining (1, 2) and (5, 4).

Solution. Slope of segment is $m_1 = \frac{4-2}{5-1} = \frac{1}{2}$. Slope of perpendicular bisector is $m_2 = -1/m_1 = -2$. Mid point is $(\frac{1+5}{2}, \frac{2+4}{2})$. So the equation can be written as $y - 3 = -2(x - 3)$. □

3.2. Exercise 1.2.23. Find the equation of the line with $x$-intercept $x = 4$ and $y$-intercept $y = 3$.

Solution. Equation of the line is $y = mx + b$, where $b$ is the $y$-intercept, hence $b = 3$. The $x$-intercept $x = 4$ will yield $y = 0$ (by definition), so substituting, we may solve for $m$. We get $0 = 4m + 3$, hence $m = -\frac{3}{4}$. So the equation can be written as $y = -\frac{3}{4}x + 3$. □

3.3. Exercise 1.2.24. A line of slope $m = 2$ passes through (1, 4). Find $y$ such that (3, $y$) lies on the line.
Solution. One way is to write down an equation of the line in point-slope form: \( y = 2(x - 1) + 4 \). Then we see clearly that if \( x = 3 \), then \( y = 8 \). Alternatively, the slope \( m \) is the change of \( y \) over the change of \( x \). Symbolically, \( m = \frac{\Delta y}{\Delta x} \) or \( \Delta y = m\Delta x \). This concept will be useful later when we deal with differentials \( dy = m\,dx \). Since the change in \( x \) is \( \Delta x = 3 - 1 = 2 \), we get that the change in \( y \) is \( \Delta y = y - 4 = 2 \cdot 2 = 4 \), hence \( y = 8 \). This method seems longer, but conceptually it is easier to do in one’s head, and will lead to intuition for calculus later.

3.4. Exercise 1.4.55. Use the addition formulae for sine and cosine to prove

\[
\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \tag{1}
\]
\[
\cot(a - b) = \frac{\cot a \cot b + 1}{\cot b - \cot a} \tag{2}
\]

Proof. Recall that

\[
\sin(a + b) = \sin a \cos b + \cos a \sin b \tag{3}
\]
\[
\cos(a + b) = \cos a \cos b - \sin a \sin b \tag{4}
\]

Now

\[
\tan(a + b) = \frac{\sin(a + b)}{\cos(a + b)} \tag{5}
\]
\[
= \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b} \tag{6}
\]
\[
= \frac{\sin a + \sin b}{\cos a + \cos b} \tag{7}
\]
\[
= \frac{\tan a + \tan b}{1 - \tan a \tan b} \tag{8}
\]

where we get from (6) to (7) by dividing top and bottom by \( \cos a \cos b \).

The case for cotangent is completely analogous. Remember \( \cot x = \frac{\cos x}{\sin x} \) and that \( \sin(-b) = -\sin(b) \) and \( \cos(-b) = \cos(b) \). Work out the details and convince yourself.

3.5. Exercise 1.4.56. Let \( \theta \) be the angle between the line \( y = mx + b \) and the \( x \)-axis. Prove that \( m = \tan \theta \).

Proof. This is trivial.

3.6. Exercise 1.4.57. Let \( L_1 \) and \( L_2 \) be the lines of slope \( m_1 \) and \( m_2 \), respectively. Show that the angle \( \theta \) between \( L_1 \) and \( L_2 \) satisfies \( \cot \theta = \frac{m_1 m_2 + 1}{m_2 - m_1} \).

Proof. This is immediate by using Exercises 55 and 56.

3.7. Exercise 1.4.58. Perpendicular Lines. Use Exercise 57 to prove that two lines with nonzero slopes \( m_1 \) and \( m_2 \) are perpendicular if and only if \( m_2 = -\frac{1}{m_1} \).

Proof. What is \( \cot(\pi/2) \)?
3.8. **Exercise 1.4.59.** Apply the double-angle formula to prove:

(a) \( \cos \frac{\pi}{8} = \frac{1}{\sqrt{2}} \sqrt{2 + \sqrt{2}}. \)

(b) \( \cos \frac{\pi}{16} = \frac{1}{\sqrt{2}} \sqrt{2 + \sqrt{2 + \sqrt{2}}}. \)

Guess the values of \( \cos \frac{\pi}{32} \) and of \( \cos \frac{\pi}{2^n} \) for all \( n \).

*Proof.* Recall \( \cos^2 t = \frac{1 + \cos(2t)}{2} \). For the general case, let \( a_0 = 0 \) and define inductively \( a_n = \sqrt{2 + a_{n-1}} \). We claim that for \( n \geq 1 \), we have \( \cos \frac{\pi}{2^n} = \frac{1}{\sqrt{2}} a_{n-1} \).

The base case is trivial. By induction, assume \( \cos \frac{\pi}{2^n} = \frac{1}{\sqrt{2}} a_{n-1} \). By the half-angle formula, we get

\[
\cos \frac{\pi}{2^{n+1}} = \sqrt{\frac{1}{2}(1 + \frac{1}{\sqrt{2}} a_{n-1})} = \sqrt{\frac{1}{4}(2 + a_{n-1})} = \frac{1}{\sqrt{2}} a_n.
\]

\( \square \)