DRAWING CIRCLES

A  (x,y)
B  (y,x)
C  (y,-x)
D  (x,-y)
E  (-x,-y)
F  (-y,-x)
G  (-y,x)
H  (-x,y)
DRAWING CIRCLES

BAD

for (x = 0; x <= r/sqrt(2.0); x++)
{
    y = sqrt(r^2-x^2);
    setPixel(x,y);
}

DRAWING CIRCLES

BAD

for (x = 0; x <= r/sqrt(2.0); x++)
{
    y = sqrt(r*r-x*x);
    setPixel(x,y);
}

REALLY BAD---SHAME ON YOU!
**DRAWING CIRCLES**

Drawing by decision

<table>
<thead>
<tr>
<th>(x,y)</th>
<th>(x+1,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(x+1,y-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE</td>
</tr>
</tbody>
</table>
DRAWING CIRCLES

Drawing by decision

M = (x+1, y-1/2)
DRAWING CIRCLES

Drawing by decision
DRAWING CIRCLES

Drawing by decision

If M inside circle choose E

x++
DRAWING CIRCLES

Drawing by decision

If M outside circle choose SE

x++
y--
DRAWING CIRCLES

Drawing by decision

\[ F(x, y) = x^2 + y^2 - r^2 \]

INSIDE if \( F(x, y) \leq 0 \) else OUTSIDE
DRAWING CIRCLES

Drawing by decision

\[ F(x,y) = x^2 + y^2 - r^2 \]

INSIDE if \( F(x,y) \leq 0 \) else OUTSIDE

\[ z_n = F(M) = F(x-1,y-1/2) = (x-1)^2 + (y-1/2)^2 - r^2 \]

\( z_n \) is a decision variable, but is costly to compute.
DRAWING CIRCLES

Drawing by decision

Rather than compute $z_n$ absolutely at each stage, compute it relatively. That is,

$$z_{n+1} = z_n + dz$$

Consider two consecutive steps:
DRAWING CIRCLES

Drawing by decision

\[ M = (x + 1, y - 0.5) \]
\[ ME = (x + 2, y - 0.5) \]
\[ MSE = (x + 2, y - 1.5) \]
DRAWING CIRCLES

Drawing by decision

Case 1: \( z_n < 0 \)

\[ z_{n+1} - z_n = F(ME) - F(M) \]
DRAWING CIRCLES

Drawing by decision

Case 1: \( z_n < 0 \)

\[
\begin{align*}
\Delta z_n &= F(ME) - F(M) \\
&= (x+2)^2 + (y-0.5)^2 - r^2 \\
&\quad - ((x+1)^2 + (y-0.5)^2 - r^2) \\
&= 2x + 3
\end{align*}
\]
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Drawing by decision

Case 2: $z_n > 0$

$$z_{n+1} - z_n = F(MSE) - F(M)$$
DRAWING CIRCLES

Drawing by decision

Case 2: \( z_n > 0 \)

\[
z_{n+1} - z_n = F(MSE) - F(M)
\]

\[
= (x+2)^2 + (y-1.5)^2 - r^2
\]

\[
- ((x+1)^2 + (y-0.5)^2 - r^2)
\]

\[
= 2x - 2y + 5
\]
DRAWING CIRCLES

Drawing by decision

SUMMARY

If z is known at step n then
{
    if (z < 0)
        z = z + (2x+3);
    else {
        z = z + (2x-2y+5);
        y--; 
    }
    x++; 
}
DRAWING CIRCLES

Drawing by decision

Getting Started

\[ x = 0; \]
\[ y = r; \]
\[ M = (1, r-0.5); \]
\[ z = F(M) = 1^2 + (r-0.5)^2 - r^2 = 5/4 - r \]
DRAWING CIRCLES

Drawing by decision

\texttt{Circle8(x,y,xcen,ycen)}
{
\texttt{x = x-xcen; y = y-ycen;}
\texttt{setPixel(x+xcen,y+ycen);}
\texttt{setPixel(y+xcen,x+ycen);}
\texttt{setPixel(y+xcen,-x+ycen);}
\texttt{setPixel(x+xcen,-y+ycen);}
\texttt{setPixel(-x+xcen,-y+ycen);}
\texttt{setPixel(-x+xcen,-y+ycen);}
\texttt{setPixel(-y+xcen,-x+ycen);}
\texttt{setPixel(-y+xcen,x+ycen);}
\texttt{setPixel(-x+xcen,y+ycen);}
}
DRAWING CIRCLES

Drawing by decision

CircleAlmost(xcen,ycen,r)
{
    x = 0; y = r; z = 5/4-r;
    Circle8(x,y,xcen,ycen);
    while (y>x) {
        if (z<0) z += 2x+3;
        else {
            z += 2x-2y+5;
            y--;
        }
        x++;
        Circle8(x,y,xcen,ycen);
    }
}
DRAWING CIRCLES

Drawing by decision

Clear Fractions

Let $h = z - 1/4$

Then $z < 0$ is the same as $h < -1/4$.

$z = 5/4 - r$ implies $h = 1 - r$ so $h$ is an integer.
DRAWING CIRCLES

Drawing by decision

Clear Fractions

Let $h = z - 1/4$

Then $z < 0$ is the same as $h < -1/4$.

$z = 5/4 - r$ implies $h = 1 - r$ so $h$ is an integer.

new $h = new z - 1/4 = z + integer -1/4$

$= h + integer$

Hence $h$ is always an integer.
But $h$ an integer and $h < -1/4$ is the same as $h < 0$.
DRAWING CIRCLES

Drawing by decision

MidpointCircle(xcen,ycen,r)
{
    x = 0; y = r; h = 1-r;
    Circle8(x,y,xcen,ycen);
    while (y>x) {
        if (h<0) h += 2x+3;
        else {
            h += 2x-2y+5;
            y--;
        }
        x++;
        Circle8(x,y,xcen,ycen);
    }
}